

Two Mathematical Patterns of Vulnerability

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Abstract

Vulnerability: an ethical category. Vulnerability must not be considered as an equivalent for the weakness or for the precariousness, i.e. the state in which the most resourceless people are living – with regard to power, wealth and health – ; the strong are also vulnerable and they are potentially likely to be weakened. We will try to give its chance to an ethical category which, unlike person, personality, dignity, benevolence, non-malevolence, focusses on pleasures and displeasures, no more on the virtues of nursing people. Moreover the category of vulnerability is quantity-sensitive. We will expose two mathematical models that highlight this last point of view.

Keywords: *vulnerability, ethical categories, Bayes, calculations, Benthamian pathology.*

“I will only know at the end what I will have lost or won in that gambling den where I will have spent some sixty years a dice cup in the hand, testeras agitans (rolling the dice).”

*Felices quibus ante annos segura malorum /
Blessed are those whose life flows away safely and gaily
Atque ignara sui per ludum elabitur aetas. /
before age’s miseries, unconscious of their own condition.
Diderot D.¹*

*‘Mille piacer’ non vagliono un tormento’
Petrarch F., Sonnet 195²*

Notwithstanding the opposition of philosophy, ‘tis certain, this circumstance has a considerable influence on the understanding, and secretly changes the authority of the same argument, according to the different times, in which it is propos’d to us.³

David Hume

¹ Denis Diderot, *Éléments de physiologie* (Paris: Oeuvres, R. Laffont, 1994), 1317.

² Quoted by A. Schopenhauer, in *Le Monde comme volonté et comme représentation* (Paris: PUF, 1996), 1338.

³ David Hume, *Treatise of Human Nature*, Book I, Part. III, sect. XIII, ed. Selby-Bigge (Oxford: Clarendon Press, 1978), 143.

At first sight, nothing is more foreign to the affective entities of pleasure, displeasure and pain than mathematical considerations, even though men have tried to compute pleasures and pains since Plato and Epicurus. Indeed, utilitarians were not the first to have that idea but they did not always quite clearly go beyond the simple ambition of computing, except maybe when the economists took over from the philosophers. I would like to see whether vulnerability, as we are approaching it, may be mathematically formalized in ways comparable to those to which the calculations of pleasures and pains, happiness and unhappiness are usually submitted. Vulnerability should be differentiated from weakness or from the frailty that results from it. Rousseau said, quite decisively, that “the strongest is never strong enough to always remain master if he does not transform his force into law and obedience into duty.” We are not going to directly adopt such a political framework as that of *The Social Contract* where this sentence comes from. Neither are we going to focus on the dressing up of force into law. Rather, we will put forward the fact that force *cannot but* be dressed up. Vulnerability is the necessity of change, its ineluctability, in the sense that whoever is assigned it does not entirely control the transformation, which is rather not in favour of the happiness, pleasure or interest of who must be subjected to it, no matter what he does. What is obvious at once is that, though there is no doubt about the fact that the weak is vulnerable (whether as regards wealth, health, power or intellectual capacities and, no doubt, many other areas), the strong may also be vulnerable. Such vulnerability may not be as directly manifest as that of the weak (precariousness), but it only takes on other forms.

From the outset, it is at least plausible that the necessary frailty of any force, whatever its degree, may be formalized in some way or other, provided vulnerability corresponds to the definition according to which the strong potentially faces a perspective of weakness and decline, which he may feel more cruelly than whoever is already weak and who, if the worst comes to the worst, has more often in himself bigger potential resources to redress the situation. The formalisations of vulnerability and those of pain and pleasure appeared approximately at the same time on two sides. *On the one hand*, under the names of *probability*, *chance* or *expectations*, mathematicians started to calculate random phenomena or events in all sorts of situations, most of the time based on the configurations of games, and even of money games. We are going to see why such a “probabilistic” model is very well adapted to a formalisation of vulnerability. Was it by pure chance that the first calculations served to model very concrete

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insurance issues?⁴ *On the other hand*, under the name of *Axioms of mental pathology*,⁵ Bentham developed the premises of a calculation, some refinements of which seem likely to be able to formalise the notion of *vulnerability*, including the idea just discovered that the strong's fear of collapse may run deeper than that of the weak. We will see how the asymmetry between pleasure and pain that the calculation allows to register makes it possible to go quite far in the formalizing of vulnerability, if such formalisation can be of any use, as we think it can.

The hidden face of probabilities could very well be another name for vulnerability

As long as one thinks that probability is an objective description of a situation and that it relates to things directly, it has no hidden face. If I have a one-in-six chance of getting an ace with a balanced dice, there is nothing hidden there, since I know I could have assigned that one-in-six chance to any other number between 2 and 6. The same applies to the division of the stakes, which Pascal deals with, and which establishes the tree of the possible paths to success or failure of each player, once the game has started and the players want to interrupt it and compensate the player who is winning. The calculation seems to be describing things and the calculation of the winning and losing combinations for each player that Fermat made underlines even more that "objective" side than that of Pascal, who assesses a situation in terms of the parts of the stakes the other players owe me or that I must give them. On the contrary, the notion of the hidden face of probability appears when one understands probability not as referring to things themselves but to a fraction of the certainty I have when I believe that such and such event will happen rather than not. If, for example, directly or based on testimonies, I have observed a certain number of times phenomena or events which are similar to A and which are subsequently related to phenomena or events which are similar to B, I estimate the chances that B might happen, given that A has happened, or even though A has not happened yet, or else, as B happens, I wonder what the chance was that A happen without me being a direct witness of it, then there is what could be called a hidden face of such type of probability based on which it is possible however to calculate as easily as on the others, though differently.

⁴ Grand pensionary De Witt won fame among mathematicians thanks to his work on the calculation of life annuity rents.

⁵ *Pathology* had not the sense it has nowadays in English of science of illnesses or troubles; it had the Greek sense of *logic of the passions*.

Bayes, an eighteenth-century mathematician and theologian who wrote *An Essay towards Solving a Problem in the Doctrine of Chances* – which was posthumously published in 1763 by one of his friends, R. Price, who was very famous among theologians, mathematicians and theoreticians of politics and politically active theoreticians – established a theorem, known as Bayes' theorem, which computes the chance I have of being right (or wrong) as I assess the probability that a subsequence of a given type may happen, in a given situation, when I know that a certain number of subsequences of the same type have occurred. Probability is a rule here in the sense that it does not describe a situation but measures an attitude, a behaviour, the beginning of a decision, which I adopt quite freely – since it does not preclude any option – and it says what chances of success I have, if I take such and such option. The example that Price takes to explain how the rule works, that of my chances of being right when I count on the sun rising again tomorrow, is not quite plausible, since, first, it puts us in a situation in which there is abundant information, all pointing to the same direction, and, second, one remains contemplative. The interest of a rule, however, is to guide us in our actions, in case subsequences are not very numerous and do not seem to point to the same direction. The rule provides the way of measuring risk assessment in given circumstances. That is why it is especially well adapted to medicine, politics, markets and everywhere one cannot but take risks, if only because one takes risks even when one does not act at all, which is a risky option the soundness of which the rule will measure. When one talks of “subjective probability” in the case of the type of problems examined by Bayes, one does not mean, of course, that all options are worth the same, and that they depend on our mood, but that the calculation assesses the probability of any option that the player chooses in whatever situation in which he has a role.

Everyone understands that it is possible to complicate the use of that rule even further by combining it with the game theory, where the object upon which I calculate chances includes a bet that the other is making or that I imagine he will make as a stakeholder in the situation. It measures through reason what I want to do in a situation in which my will is an accomplice of other wills, or is opposed to them, which are acting in their interest or in interests that, if I want my action to succeed, I had better identify.

If we think that that rule expresses *vulnerability*, it is quite exactly because it stages risk-taking, whether big or small, and shows its chances of success provided what happens is in keeping with what I have calculated or wanted to happen, as well as establishes the chances of failure if I choose a risky option. Any action I

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decide or about which I think, may fail, and the margin that is left to failure, and against which I cannot do anything, is the limit of my control, and may be called *vulnerability*. Nothing may be done without the possibility that what I do not want to happen happens indeed, and nobody is ever in a situation in which they do not want anything.⁶ Doing nothing is wanting to do nothing, or wanting to let things happen. The necessity for one to act, together with the freedom of choosing the content of one's actions and the account reality holds us to constitute the very structure of vulnerability.

The rule happens in an uncertain world, which has no absolutely stable rules, where rules are crumbly, as Price shows in his astronomical example – the surest rules are still subjected to probability. It is such that the decision that has been taken, based on the information I had on the development of the situation at a given moment, may be good, even though the evolution of the situation seems to reveal (or will reveal) that my decision was in reality bad or even disastrous. The true and the false are not overhanging the situation in such a way that it cannot be assessed from an outside point of view. There is no exteriority of the situation in which I have to act and take decisions.⁷ Not only do they not overhang the situation, but they belong to it in order to constitute it, that is, to become the elements of the situation that participate in its evolution and in the building of a development or an outcome.

The name *hidden face* may be quite deceptive, for it implies that there is a position from which our situation would be fully revealed although no transcendence other than a fantasmatic one is possible there. Such ghostly transcendence may be seen even in scientists and even in the most atheist of them. In a way, though it is an unavoidable illusion, it has no place in probability calculations, which do not need to lean on an intelligence which would know everything at a given time or on the supposedly full knowledge of a situation, despite what Laplace and Kant said, though with a different meaning. It would be better to speak of an *uncontrollable other side* of the probability I assign to one of the projects I undertake. I am referring here to the uncontrollable residual part that my decisions can constitute, but without knowing how they constitute it, for there

⁶ This formulation may be considered as the generalization of the famous fragment of the *Pensées*: “We are so unhappy that we can only enjoy something which we should be annoyed to see go wrong” (Laf. 56, Br. 181, B. Pascal, *Pensées*, trans. J. Krailsheimer (London, New York, Victoria: Penguin Books, 1995).

⁷ That is the point that Kant does not take into account in probability. There is no sign of Bayesian probability in his work. He does not think that there can be any logic in probability and, quite strangely, calls mathematical *probability* only a probability over 1/2.

is always a distance between what I contemplate and think I want in an action – the objective I think I want to reach – and the result I get thanks to it without really knowing how I have got it, and which is nonetheless reality itself.

Hence the tale-like structure of probability problems, which, though formal, tell a story that players enact, while telling themselves another one. Such a constant misunderstanding among players on the one hand and between the players and the situation on the other hand, constitutes and tells the story of vulnerability. Bayes' *Essay* tells a story like the texts which explain the most formal aspects of the perspective seem to give a classical overview, while doing nothing else than telling mathematical patterns or letting them express themselves.

We will have to criticize those patterns, but first we must say why we are talking of schemas, which will be more obvious thanks to the analysis of the second example.

An axiom of Benthamian pathology

In a set of axioms which come after his *Pannomion*⁸ and which were written during his whole life, since a few of them date from June 1831 – Bentham died in June 1832 –, there is a strange remark upon which we would like to draw attention.⁹ Bentham shows that we are more unhappy when we lose something that is dear to us than we are happy when we win something that, for us, has the same value. Values being equal, it is more painful to lose something than pleasurable to win it. Of such a statement, which agrees with intuition, Bentham provides a more symbolic demonstration – a *figuration* would be the most accurate word to call it – which is of interest for us here.

Indeed, if I own an S sum and I lose a sum that is smaller than S which I will call DS , I cannot but relate that DS to the sum that remains and which will be written $(S - DS)$. On the contrary, when I win the same sum DS , and I cannot but relate it to the sum I now own, and which can therefore be represented by $(S + DS)$, I note that the relation which signifies my sadness, my disappointment or my disagreement, i.e. $[DS / (S - DS)]$, is higher than the relation that is supposed to express my joy, my happiness, or my pleasure at winning, i.e. $[DS / (S + DS)]$. Here is the form that expresses and even makes explicit the rationality of the axioms of pathology stated by Bentham and which I first enounced in the vernacular language to more directly address intuition. We have just demonstrated

⁸ The complete Code of laws that Bentham has throughout his life wanted to write.

⁹ *The Works of Jeremy Bentham*, éd. J. Bowring, vol. III (Tait, Edinburgh; Simpkin, Marshall, Londres, 1848), 224-230. That set of axioms is part of what is called the *Pannomial Fragments*.

thanks to a slightly formalized structure why an affect of disagreement or physical pain is, all other things remaining equal, more painful, more perceptible, more acute than an affect that is triggered by an object having the same value is agreeable.¹⁰ Such asymmetry according to which our pleasures are less perceptible than our despairs explains – at least partly – the reason for vulnerability and goes beyond a simple factual remark of a psychological character as may be found in Hume’s *Dialogues on Natural Religion* which supposedly shows there is more evil than good in the world, which results in damaging the idea of a divine Providence that provides men with happiness.¹¹ Men would necessarily be less disposed towards pleasure and happiness than towards displeasure and unhappiness, and the fate of happiness would more often be to worsen into unhappiness than the fate of unhappiness to reverse into happiness.

The subtlety of such a calculation, of which those are only the starting points, is due to the integration, in the very calculation of objective elements – production, exchange, consumption, which may be placed under the sign of interest¹² –, of passionate elements, which are usually believed to be more

¹⁰ To be more thorough, Bentham could have specified that there is no way of escaping this rule, since to experience pleasure, one must risk displeasure and pain, as Pascal very well saw, “We are so unhappy that we can only enjoy something which we should be annoyed to see go wrong” (Laf. 56; Br. 181; Pascal, *Pensées*).

¹¹ In Part X of *Dialogues concerning Natural Religion* which, together with part XI, makes a terrible list of all the evils that affect our lives, Philo immediately answers to Cleanthes who suggests that “health is more common than sickness; pleasure than pain; happiness than misery. And for one vexation which we meet with, we attain, upon computation, a hundred enjoyments” (ed. R. H. Popkin, Indianapolis: Hackett Pub. Company, 1986), 65: “[Your position] is extremely doubtful, replied Philo; you must, at the same time, allow, that, if pain is less frequent than pleasure, it is infinitely more violent and durable. One hour of pain is often able to outweigh a day, a week, a month of our common insipid enjoyments; and how many days, weeks, and months are passed by several in the most acute torments? Pleasure, scarcely in one instance, is able to reach ecstasy and rapture; and in no one instance can it continue for any time at its highest pitch and altitude. The spirits evaporate, the nerves relax, the fabric is disordered, and the enjoyment quickly degenerates into fatigue and uneasiness. But pain often – good, God, how often! – rises to torture and agony; and the longer it continues it becomes still more genuine agony and torture. Patience is exhausted, courage languishes, melancholy seizes us, and nothing terminates our misery but the removal of its cause – or another event, which is the sole cure of all evil, but which, from our natural folly, we regard with still greater horror and consternation.” (Hume, *Dialogues concerning Natural Religion*) A simple mathematical expression allows to at once, at least empirically, understand the necessity of that asymmetry. Schopenhauer never forgot those pages and echoed them in *The World as Will and Representation*, §§ 56-59 Book I, Chapter XLVI, entitled “On the Vanity and Suffering of Life” (Paris: PUF, 1966), 1345. In that text in particular, he emphasizes the fact that “Only suffering and deprivation can produce a positive impression and thereby reveal themselves: well-being, on the contrary, is nothing but pure negation” and “any pleasure is always reduced to half a pleasure” (Schopenhauer, “On the Vanity and Suffering of Life”, 1337, 1340).

¹² In Hirschmanian terms.

“subjective”, but which in reality may, as easily as the former, be included into equations.¹³ Even though Bentham distinguished passions from what would mainly motivate our behaviours, and even though his thoughts were more in line with philosophies of interest rather than of passions, he did not forget passions in his calculations and radically tied them to deeper motives.

However, the difficulty of this argument is directly linked to the fact that one can find an interpretation of it that will make it express on the whole the opposite. Let us suppose that, among an ocean of troubles, which I will call S, I can benefit from the solution to one of my troubles, noted DS. The schema of the operation could be written $[DS / (S - DS)]$, since a trouble is removed. Conversely, if to a crowd of troubles S another is added DS, the schema of the operation can be expressed by $[DS / (S + DS)]$. The pattern which expresses relief is noted by a relation that is bigger than that which is supposed to represent the sinking into troubles or despair. Thus, the result that is obtained is the reverse of the previous one, which nonetheless seemed well founded and in keeping with the often noted intuition that an accumulation of disasters drives someone less to despair than to laugh. It is as if one could not believe it.¹⁴ Our confrontations of relations therefore do not directly express, thanks to their mathematical appearance, that, values being equal, happiness is more elusive than unhappiness.

Does that completely discredit any attempt at giving affective processes a mathematical turn, at least in the direction that Bentham pointed to, and consequently the formalisation of vulnerability that we are trying to conduct? We do not believe it does, but it is necessary to explain why on two levels. The *first* allows us to explain why we should talk of an *explicative pattern*. Perhaps would it be preferable to speak, like Kant, of explicative *schemata*.¹⁵ A *schema* is a sort of intermediary between what seems given by experience and the concepts thanks to which we want to think about that experience. A schema cannot find its own direction without a methodical idea supervising it, such precedence being exactly the function of the concept and of the judgment that uses it. In other words, the precious moment of the *schema*, the essence of which we borrow from Bentham and the name from Kant, is the representation of a necessary asymmetry between

¹³ Their fate is the same as that of those “small equations” which Laplace mentions to better adjust the ideal laws of Newton which globally work with what experience and observations make us modify.

¹⁴ Valéry notes it in his *Cahiers*, vol. II (Bib. de la Pléiade, NRF, 1974), 466: “Accumulated catastrophes make one laugh [...]. They lose any seriousness. Sensitivity gives in and reverses.”

¹⁵ I. Kant, *Critique of Pure Reason*, trans. ed. P. Guyer & A. W. Wood (New York, Melbourne, Madrid: Cambridge University Press, Cambridge, 2000), 273.

pleasant and unpleasant elements, which is often noted, without any explanation of its motive being provided. Expressing something in mathematical terms, even broad ones, as we have just done, allows to make this “explicative engine” work, whatever its misfirings and malfunctions. The *second register* refers to the management of the small mathematical element we have just used. The affective dimension the symbolical meaning of which we have just expressed is not directly in harmony with the operation and development of what triggers it; it is in a relation with it, but going another way, it is out of sync with it, which does not mean that the one can work without the other. One could say, to use Hume’s words – Hume described the phenomenon of passions thanks to a double association – that the affective accompaniment of real, economic and political acts has its own reality, and that it is not only a reflection of it. That out-of-sync relation is what is of interest and what mathematics allows us to express, provided it is supported by a clever method.

One can understand for example that a fiscal policy of redistribution of the surpluses of the richest to the poorest is undoubtedly the best interest in democracy, at least what is best in line with its egalitarian dimension. A democracy is in danger when extremes, as far as wealth, power or advantages are concerned, move in opposite directions.¹⁶ One could then be tempted, if one had some political power and really wanted to make democracy happen, in a realistic and pragmatic way, to abruptly organise the economic equality of citizens. One would nonetheless be very likely to politically fail if one suddenly and brutally imposed such a measure. Though one should tend towards such equality, the realization of it must be negotiated so as to hurt as little as possible those who have to or must give money to the poorest, when one has realized that it is harder to give money than it is pleasant to receive it. One must make sure not to offend or drive to despair those who, in a situation, have a lot to lose, and are therefore the easiest to be upset, all the more so as in a democracy in particular, there will be no great gain, no electoral gain for example, for whoever takes that measure, from those who are never more than moderately satisfied.¹⁷ The laws of passions are not

¹⁶ Rousseau clearly highlighted the point at the end of the first Book of *Social Contract*.

¹⁷ Taking a more general example, Hirschman notes that if one were tempted, in order to determine the interest of interstate relations, to take into consideration only the trade balance, one would be quite mistaken, for there are beneficiaries on the two sides: “Many effects [that the parties expect from the exchanges] are political, social and even moral rather than purely economic” (<https://books.google.fr>) (*Les passions et les intérêts / The Passions and the Interests*, 51). That idea was supported, for example, by Montesquieu, quoted by Hirschman, *The Passions and the Interests*, 75.

those of interests. One should not mistake the one for the other, even though one should listen to each equally.¹⁸

Bentham was so attentive to that point that he renewed such advice about sharing power and not only wealth. It is undoubtedly sound to throw out those who have not been able to conduct the policy that would be the most useful to the greatest number, but there is no need either to offend the leavers by brutally depriving them of what was, until now, their livelihood. The unpleasantness of their departure should be smoothed out, in the name, once again, of the very axiom that must guide practice. The examples we have just given, though basic, show that the line of utility¹⁹ which must run along all the political, administrative and economic acts,²⁰ in reality results from the intertwining or twisting of two lines, one that would readily be attributed to ‘reality’ and the other to the affects, which obey other laws, and which one would be wrong to think are irrational or insignificant in any given situation. Affects are as important as what we have called the things themselves, and neglecting them and taking only things into consideration would be a very poor job indeed. Stuart Mill followed Bentham on that point, and put on trial the revolutionaries and reformers who, without any qualification whatsoever, wanted to impose what they thought was fair and severely punish those who had not fulfilled their function properly or had carried out a policy that was different from their own.

¹⁸ A. O. Hirschman, in *The Passions and the Interests*, showed that there should be no mixing up of the one and the other; and quoting from Cardinal Retz, he underlined that “a truly subtle politician does not wholly reject the conjectures which one can derive from man’s passions, for passions sometimes enter rather openly into, and almost always manage to affect unconsciously, the motives that propel the most important affairs of state” (Princeton, New Jersey: Princeton University Press, 1977), 45. (That excerpt from the *Mémoires* (Paris: Gallimard, Bib. de la Pléiade, NRF, 1956), 1008-1009, is quoted with a footnote in the same page: “Elsewhere Retz writes similarly: «In the times... in which we live one must join the inclinations of men with their interests and draw on this mixture in order to make a judgment on their probable behavior.»” Cardinal Retz, *Mémoires*, 984. A strikingly similar opinion is expressed over a century later by Alexander Hamilton, another practicing (and reflective) politician: «Though nations, in the main, are governed by what they suppose their interest, he must be imperfectly versed in human nature who... does not know that [kind or unkind] dispositions may insensibly mould or bias the views of self-interest.» Cited in Gerald Stourzh, *Alexander Hamilton and the Idea of Republican Government* (Stanford, Calif.: Stanford University Press, 1970), 92.”

¹⁹ This is Bentham’s word. The criticism of Article 6 of the 1791 Human Rights mentions the “curves and bends of the line of utility, which must be the sole object of the legislator” (Bertrand Binoche and Jean-Pierre Cléro, *Bentham contre les droits de l’homme* (Paris: PUF, 2007), 55. Of course such a line does not exist substantially speaking: it exists only as the law of the work of each stone in stereonomy.

²⁰ Just as the fictitious curves that run in the buildings that have been thought, in stereonomy, by Bosse and Desargues, and which are like the reason of their walls, windows, vaults, arches and roof.

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It is obvious here that it is not because utilitarianism considers affects as fictitious entities that it does not take them into account. They simply do not have the same role, and do not work, in its calculations, like the productions, exchanges or consumptions they go with. The concern for the greatest happiness of the greatest number is not necessarily related to disdain for the pain of the individuals it affects, even though they be minorities. The fall from grace of the powerful or of the one who used to be powerful should be arranged so that it is not too violent. Vulnerability therefore is expressed in those lines which act as a counterpoint, or, to use another metaphor, act the one towards the other as a double helix. Calculation is the connection of those two cursive lines, one of them representing the marked tendency of passions to become sad or disagreeable, to burden the actor. It is up to the politician, the administrator, the businessman or the doctor²¹ to be subtle enough to play upon that double helix when he undoubtedly has had the opportunity – and has seized it – to play upon it, or to have free rein to play upon it, knowing that one must act if one wants to somehow check the fate of affects which is overall to privilege displeasures over pleasures. There is undoubtedly, in the intertwining of voices, be they of reality or of affects, matter for stories which invent characters rather than events happening to ready-made characters. It is not

²¹ “Arithmetic and medicine – these are the branches of art and science to which, in so far as the maximum of happiness is the object of his endeavours, the legislator must look for his means of operation: – the pains or losses of pleasure produced by a maleficent act correspond to the symptoms produced by a disease.

Experience, observation, and experiment – these are the foundations of a well-grounded medical practice: experience, observation and experiment – such are the foundations of a well-grounded legislative practice.

In the case of both functionaries, that of the *medical curator* and that of the legislator –, the subject-matter of operation and the plan of operation is accordingly the same – the points of difference these: – In the case of the medical curator, the only individual who is the subject-matter of the operations performed by him, is the individual whose sufferings are in question, to whom relief is to be administered. In the case of the legislator, there are no limits to the description of the persons to whom it may happen to be the subject-matter of the operations performed by him.

By the medical curator no power is possessed other than that which is given either by the patient himself, or in the case of his inability, by those to whose management it happens to him to be subject: – by the legislative curator, power is possessed applicable to all persons, without exception, within his field of service; each person being considered in his opposite capacities – namely, that of a person *by whom* pleasure or pain, or both, may be experienced, and that of a person *at whose hands* pleasure or pain, or both, may be experienced.” (*The Works of Jeremy Bentham*, ed. J. Bowring, vol. III (Tait, Édimbourg; Simpkin, Marshall, Londres, 1848), 224.

Note that, in that preamble to the discourse on axioms of mental pathology, it is less the money that has been won and lost which serves as a privileged pattern than health that can be preserved, recovered or lost. Simply, calculations would be impossible if, instead of using money, the universal mediator, one used health and its disruptions. The latter must nonetheless be thought of behind the calculations that are being made with money.

because the story has a mathematical appearance that it is not a story. There exist stories outside literature. There is, in problems and their solutions, a tension that belongs to stories, and a problem which does not “tell” any story is not interesting. There are, in mathematics, accompanying curves, fictitious lines of all sorts, as in architecture or music, where, without being materialized by any note, a median voice can play its part.

There is so little contradiction between the calculation and the language that literature is interested in, that such a double helix we have mentioned could easily be compared to a phenomenon of echo, the one being built as an echo to the other. Maybe echo is the essence of language, in the sense that, without it, speech would always seem to be emitted by a subject and would not appear to be what belongs to no subject in particular, being thus more objective. Like the poet and the architect, the politician, the administrator and the businessman have to build that echo, which is however what is the least controllable.

Obstacles to the previous calculations

However, one’s enthusiasm being dampened by the few counter examples I have given to the double association of the affects and elements of political economy, one could think that such a mathematics of vulnerability is only imaginary and desired, that it only puts into formulas or even only gives the idea of formulas of what will never have even the appearance of a fragment of science. Undoubtedly, economists have tried that conceptual polyphony, Albert Otto Hirschman in particular.²² However, we must if not make a detailed inventory of all the obstacles that must be overcome to reach such a result, at least give a short list of them and ask ourselves if in principle, there is a decisive obstacle to the mathematical formulation of that double associative helix. There seems to be one, and quite a decisive one, which Kierkegaard spotted, and which is a jeweller’s scale, for Bayesian and utilitarian approaches, in which the author of *Philosophical fragments* was not however directly interested, and by which previous research is partly compromised, which is not a real problem in itself since there are always only partial solutions in mathematics as elsewhere, but which nonetheless is worth our stopping a little to examine them.

²² Hirschman, the author of *The Passions and the Interests*, subtitled *Political Arguments for Capitalism before Its Triumph*, who died in 2012, wrote in that book in 1977: “Cardinal de Retz, with his insistence that the passions are not to be counted out in situations where interest-motivated behavior is considered to be the rule, appears to have had the better part of the argument than either Keynes or Schumpeter” (Hirschman, *The Passions and the Interests*, 135).

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The ordinary arguments, which are here only quickly listed, and which can be opposed to a mathematization of an affective principle like vulnerability, were extensively developed by Bergson who, at the end of the 19th century, questioned Fechner's psychophysiology which quantitatively treated sensations and the excitations that triggered them, and questioned the fact that the ones may be used in the same equations as the others. It is easy to answer, first, that it is not absurd to say that a sensation can be felt *more or less* strongly, and, second, that putting sensations and excitations in the same equation does not mean that they are being treated the same way. Moreover, let us not forget that the object of psychophysiology is rather to speak of a fictitious space, of a difference, between sensations, rather than of sensations in a substantial sense. What psychophysiology talks about in mathematical terms has no direct correspondent in experience and relates to it only via fictions. It is quite obvious that by attacking Fechner, Bergson indirectly attacked Bentham's utilitarianism, which, it is true, is not one of the targets of *Essay on the Immediate Data of Consciousness*, but which was on the contrary deliberately mentioned in his lessons at Clermont-Ferrand or Henri IV high schools, which were delivered at the same time as he wrote his PhD.²³ Bergson's arguments, or at least their critical part, must have made quite an impression since some authors, who are little inclined to philosophize about "duration", renounce a cardinal treatment of sensations and affective experiences and prefer an ordinal treatment of pleasures and pains, in all likelihood to avoid the criticism of treating felt entities as quantifiable things. That was the case of George Moore, who, in *Ethics*, ordered pleasures rather than add, subtract, multiply or divide them. It is quite easy to show however that it was not so difficult to substitute to such a little practicable order a cardinal interpretation, which does not commit a direct realistic ontology and which is easier to treat, in economics and in other human sciences, than preference orders. Using the same style of opposition, some authors have insisted on the irreducible difference between "feeling" a sentiment and the logical or mathematical structure that would be used to account for it. It is true that it is not difficult to agree to that provided the criticism of the phenomenologists of affectivity does not go beyond negating the opposite party who supports the possibility of identifying – up to a certain point – a sentiment to a structure or to a situation, for as soon as it posits its own difference with the point of view of structures, it only yields very plain and poor tautologies, which are so devoid of interest that they immediately make one regret structural analyses which

²³ Henri Bergson, *Cours*, I, Clermont-Ferrand, 1887-1888 (Paris: PUF, 1990), 61-64; 163; *Cours*, II, Lycée Henri IV, 1891-1893 (Paris: PUF, 1992), 59-76.

have at least the merit to put on the same level the analyses of affects and those of some human sciences like economics, sociology, ethnology, or history. It is only because one is prejudiced that one thinks that vulnerability focuses on what is the most “felt”, of the least “structural”, of the most “individual”, “ordinal”, of the least “cardinal”. The two examples we have chosen already allow some circulation between categories that counterbalance that prejudice, all the more so if we can invent the mathematical form that best suits our needs.

If one looks at less massive criticisms of the mathematization of issues of affects, then one faces the objection of a reduction of passions, and pleasures and pains especially that are supposed to underlie them, to situations of gains and losses of money, which happen in a general game pattern. When one thus transforms money and money games into paradigms of what happens in real affective exchanges – provided such exchanges are real – does not one distort the analysis one wants to subject them to? It is clear that the attitude one has when one plays is not the attitude one would have in a real situation, that one takes, for example, more risks in a game where the stakes are strictly limited than in reality when no limit is set from the start. It is no less clear that if money allows to avoid focusing on falsely isolated subjects, even though it realizes some intersubjectivity, which itself is problematic but in a different way, it would not be possible to account, through it, for all the affective situations. However, through the representation by money, what is symbolized is the gain and the loss, which have a real affective meaning.²⁴ Thus, when one considers such – more or less fictitious – use of money and that of the privileged sequences of the game theory as paradigms in order to think about affectivity or one of its principles, such as vulnerability, one has what is called a *pattern* or a *schema*, that is, a tool that should not be applied with systematic blindness which would exempt one from using any method, but, which, on the contrary, requires a discerning judgment, that is, the consciousness of the limits of the use of the tool in “given” circumstances.

²⁴ We do not have enough time here to consider a point that derives from what Hirschman saw in *Les passions et les intérêts / The Passions and the Interests*: in order for money to be called thus to measure passions and passions and interests, it has had to be rehabilitated and – as Hirschman quoted from Hume (*Les passions et les intérêts*, 64) – it must be a time when love of gain is deeper than love of pleasure, which is the essence of capitalism and of the “enjoyment” of its added value (of its “more pleasure”, as Lacan would say), or at least the time must be that of those who think thus and have the power to impose that rule on the others.

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The main obstacle: the impossibility to think the “decisive instant”

Had we managed to answer, somewhat satisfactorily, all the previous objections against mathematizing methods which we think could be implemented, there would still remain a difficulty which might not be foreign to those that have been raised and which is related to the insufficient, or even non-existent, treatment of the *instant* by the mathematically-inspired methods we are interested in to deal with vulnerability, and which has appeared as the necessity of the decline in the forces – including the strongest – as well as that which could check that decline if they want to remain forces. Such ultimate objection can be found, as we have said, in Kierkegaard’s *Philosophical Fragments* which oppose the Greek incapacity to *think the instant* and the modern demand to put it at the core of ethical systems, which may not be done through mathematics.

Russell dared to say that no one had ever answered Zeno’s paradox – in particular the fourth of them, based on Aristotle’s account – which makes it impossible to seize the instant when movements cross one another, that we are still, as regards that instant, *either* in the presages that prepare it, *or* in the aftermath;²⁵ what is missing most is the *decisive instant* as Kierkegaard called it in a thesis in which, strangely enough, he agreed, at least on that point, with the great logician who nonetheless had no particular esteem for him.²⁶ What was Kierkegaard’s objection towards the ethics of his time²⁷ – which is still ours overall – and which reveals in them the greatest difficulty?

The following: That ethics, be it Bayesian, or utilitarian in its critical part, is never in the end about the game of deliberations, but leaves aside the instant of the decision which works as a blank in this game. All the philosophies of ethics only

²⁵ After analyzing the three main paradoxes of Zeno that are in Aristotle’s *Physics* and which he thought were not solved before Weierstrass, Dedekind and Cantor, Russell analyzed more rapidly the fourth and took Zeno’s side, refusing to suppose that, when there is a movement, the mobile “will be in a neighbouring place at the next instant”, simply because “there is no next instant”. “When a body moves, all that can be said is that it is in one place at one time and in another at another”. “Even though philosophers often tell us that when a body is in motion, it changes its position within the instant”, “to this view Zeno long ago made the fatal retort that every body always is where it is; but a retort so simple and brief was not of the kind to which philosophers are accustomed to give weight, and they have continued down to our own day to repeat the same phrases which roused the Eleatic’s destructive ardour” (B. Russell, *Recent Works on the Principles of Mathematics*, 1901, in: *The Collected Papers of Bertrand Russell*, ed. G. H. Moore vol. 3 (Routledge, London, 1994), 371-2. Zeno’s fourth argument is better analyzed in § 334 of *The Principles of Mathematics* (Routledge, Londres, 1992), 352. In that paragraph, the “continuity” of the philosophers is refuted. Curiously enough, Kierkegaard should not be counted among them.

²⁶ There is not the slightest trace of Kierkegaard in Russell’s *History of Western Philosophy* nor even, I think in the whole of his published work.

²⁷ Which he links all to socialism, even though they have explicitly denied it.

manage to be ethics of deliberation. They never reach a thinking of decision, except for that which Kierkegaard thought of, and those which take Zeno's arguments seriously. Even those who praise "decision makers" show, by creating that word, which seems to designate indifferently a function, a position, or a character feature, that they do not understand what is at stake in a decision. The time of decision is that of an instant that has nothing to do any more with that which was dreamt of so much as long as it still had to be overcome, and which will be referred to once it has passed without our having time to seize it. Abortion is not reasoned about, when it is not here yet or when it has occurred, in the same way as at the moment when one acts and performs it. One does not get a patient to sleep definitely, even though one has taken every precaution regarding the consent of the interested party, as one would discuss the merits of that sedation before one performs it, as one will refer to it after the patient has died. One does not talk of the value of a life in the same way as one takes away that life. The medical act is intertwined with instants in which all the construction of time perspectives that some mathematics allows – and which we have just started to get a glimpse at – is disrupted and invalidated. It is impossible that the measure be the same at the moment when one kills someone and when their death is only being considered. In order to point out to a problem, more than to solve it, Kierkegaard called that time *instant*,²⁸ to which he added the adjective "*decisive*" in *Philosophical Fragments*.

The real ethics should start, for him, from that instant, which cannot be compared to any other, for it is not possible to represent it, unlike the others, and even to think about it. Kierkegaard pointed to the place where to look deeper into the ethical question.²⁹ Maybe he even showed the equivalent of a topology of it, but then, having only communicated the intuition of it, provided it can be communicated, has he shown what its structure could be?

Our analogy with the perspective has but a very partial value. We know the rules of perspective. They give us, when they are expressed in a painting, the illusion that the painting, drawn or painted from a point of view that it imposes upon us, could have been made from a thousand other points of view which are

²⁸ "And now, the instant. Such an instant is of a particular nature. Undoubtedly it is short and fleeting, as is any instant, passing as any other into the following instant. Nonetheless, it is the decisive one, nonetheless, it is full of eternity. Such an instant must have its own name. Let us call it the fullness of times" (S. Kierkegaard, *Les Miettes philosophiques* (Paris: Éditions du Seuil, 1967), 50.

²⁹ "The instant in time must have a decisive importance, so that in no point of time and eternity I could forget it" (*Riens philosophiques*, trad. K. Ferlov, J. Gateau (Paris: Gallimard NRF, 1948), 54) or, later on, "For a temporal starting point not to be nothing, the instant in time must have a decisive importance."

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contained as *possibles* in it. However, such *possibles*, far from being able to be exteriorized, have no meaning other than based on the painting as it is, in its partiality, and not the reverse, although it presents itself as a possible that has been chosen among a huge range of other possibles. A possible has been chosen and it exists in a sudden way. *Mutatis mutandis*, the same is true for time, as for space, with the crucial difference that if, when looking at the painting, one can *reproduce* the crucial and lightning point of view where it is supposed to locate us to look at what it represents, one could not give back the precise instant of will. The seriousness of life does not repeat itself: it “gets ahead of itself” or glides into the “aftermath” which – with prevision – does not have the same properties as the fleeting instant which is impossible to set. The construction of temporal perspectives, which the strategies of deliberation and the consequences one gets from the advent of a decision are, is much more fictitious than that of space perspectives, for the real entity that is being referred to to anchor it or to found it has much less reality. It is as if the reality it is possible to “recapture” in space emptied at once by briefly appearing in time. The instant is the abyss of all deliberating constructions.

Conclusions

Kierkegaard, in *Philosophical Fragments*, opposed the point of view of the Greeks, whom he accused of not knowing or omitting the *instant* and of neglecting its precious and frail ethical value, and the modern point of view which he defended by stressing the instant,³⁰ which, for us, does not have its philosophy or its ethics yet, except perhaps in the Old and New Testaments. Antinomy is of a huge interest. However, presented thus historically, it is debatable. Because the Greeks were conscious of it even though they could not – despite their mathematical and logical skills – remedy that issue. There is no Zenonian ethics that I am aware of, but one could not reproach Zeno for not seeing where the difficulty was. However, to move onto the opposite side of the Greek thesis, the

³⁰ “So if the *instant* must have a decisive signification, without which we would only speak Socrates’ language, whatever we say, and how strange soever our prolixity be, and even though as we do not understand ourselves, we imagined that we went much farther than that wiseman who is so simple, the judge and incorruptible arbiter of the god, men and himself [...] thus the break happened, and man cannot go back and will not find any joy in recalling what his memory presents him with, and even less will he be capable, with his own forces, to draw the god to his side again.” Kierkegaard, *Les miettes philosophiques*, 52. And, later on, “While all the pathos of the Greek thought is focused on memory, the pathos of our project focuses on the instant” (Kierkegaard, *Les miettes philosophiques*, 53).

Hebrew Bible and the New Testament do think about the instant,³¹ but have they been more able to produce an ethics? I am not talking about morals, since morals do not need to be expressed, whereas it is as much an imperative duty for ethics as for the law that cannot but be enounced. Has Kierkegaard gone beyond the intuition of a science that must still be built, if that is possible?

One could even push one's suspicion even further: Is not such a way of pointing to the *instant* as an ethical requirement, under the pretext of moving on to a religious level, a pure and simple return, beyond ethics of the interest and the useful, to a temporality of passions – of some of them at least, if one admits that they are not all of them peaceful ones and that there may be some violent ones? Who does not know that it is possible, in one instant, in one look, in one word – good or bad – for a meeting, to reverse the perspective of a whole life or of several decades? Do anger and love operate as boredom or hope, which can accommodate to the longtime of calculations?

Kierkegaard is nonetheless right, in an apagogic way, against the thesis he attacks at its weak point: the requirement to say, the patterns of which can be enounced in mathematical terms, can only have an application during the time of deliberation, which is almost all the time, if one thinks of it. But there only needs, within that wholeness, one instant for the whole building to become suspicious, for no one knows what the decision is, what it will be or was, especially if one has not taken it oneself, and even if one had to take it or had taken it, for it leaves no other trace than the need to justify oneself or to forget, than the happiness or misfortune to have ushered in a new world. It is difficult to have, in mathematics, or in the *langage* of calculations, another discourse than that of the preparation and that of the assessment of what has changed. The discourse of the double helix, one movement being the echo of the other,³² that we have had, allows nonetheless to explain how mathematics can eclipse the crossing moment the elision of which was an object of astonishment for Zeno, and can give it the status of an illusion, as if it had never existed and never could: lines can never cross, nor movements contact, even though there were no point of view, no perspective, no projection that would allow one to make them appear as not crossing or contacting.

The instant of the decision being removed, which is a disastrous elision for ethics, does not however affect the possibility of the discourse on pains and sufferings, passions, interests and preferences to be on a par with the calculations

³¹ The archetype of that collusion of the instant of death and the instant of the decision is, obviously, in Kierkegaard's reflection, what Christianity has called "Abraham's sacrifice".

³² As when a voice seems to be gone backwards by its double.

of the economist and the politician to whom it gives another line of readability. That knowledge could very well be radically oriented towards practice: they are nonetheless as theoretical as mathematics or physics, since they are not concerned with Kierkegaard's *instant* and are defenceless faced with the suddenness of decisive events they leave unthought when they are supposed to happen. That is precisely why a research of the mathematical objects of vulnerability can be of interest. Far from it being inhuman to calculate, including affective questions when what is at stake is to compare them and to relate them to different economic fields, which are so important for ethics and for the sciences it needs, one must recall that the person who wrote on the pediment of his school that "nobody may enter unless he is a mathematician" dared affirm in *Epinomis* that we are being more fundamentally made for *numbers* than for *language*.³³

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³³ "If one withdrew number from mankind, one could never show any wisdom. Never again, the soul of that living being that man is, would acquire the virtue as a whole, given that he would lack reason." "If we should deprive human nature of number we should never attain to any understanding. For then the soul of that creature which could not *tell* things would never any more be able, one may say, to attain virtue in entirety; and the creature that did not know two and three, or odd and even, and was completely ignorant of number, could never clearly *tell* of things about which it had only acquired sensations and memories. From the attainment of ordinary virtue – courage and temperance – it is certainly not debarred: but if a man is deprived of true telling he can never become wise, and he who has not the acquirement of wisdom – the greatest part of virtue as a whole – can no more achieve the perfect goodness which may make him happy. Thus it is absolutely necessary to postulate number; but to show why this is necessary would need a still further argument than any that has been advanced" ("Epinomis," 977 c, in *Plato in twelve volumes*, XII (Cambridge (Massachusetts), Heinemann W., London: Harvard University Press, 1986), 439-441.

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