Modal Logic and Its Forms

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Abstract

The discursiveness in the process of (logical) argumentation must take into consideration a series of aspects connected to the concepts used, the definitions resulted with the help of these concepts, the theories analyzed. From this perspective, the modal logic and its specific forms involve a reevaluation of reasoning. Moreover, through pertinent discursive argumentations are visible problematical approaches which have certain philosophical significances. Therefore, the new logics reflect at the level of discursiveness a certain meaning which participates in its turn in assuming a certain logical paradigm.

Keywords: *the modal logic, the temporal logic, the epistemic logic, the logic of action, the logic of acceptance, the dynamic logic, the teleological logic.*

1.1 General characterization of the modal logic

The modal logic (founded in fact by Aristotle[1]) represents that part of logic where the modal propositions are studied. The modal propositions are formulated like this: "It is necessary to learn the modal logic", "It is possible for us to go on a trip tomorrow", "It is forbidden to walk on this road" etc. In these sentences one can notice expressions such as: "it is necessary to", "it is possible to", "it is forbidden to" which are the subject matter of modal logic. Still along these there are other expressions or notions specific to this type of logic: "contingent", "existent", "falsified", "logical-false", "factual-true", "obligatory", "permitted", "good", "worse", "always", "later" etc. All these expressions (notions) belong to the modal logic especially, and if they are analyzed privately, they belong to subfields ("private logics") of the modal logic. In other words, in modal logic are used modalities that relate to propositions, events or certain states of facts. "The modal logics are generally built as superior levels of the logic of propositions and the logic of predicates. The logic of prepositions, its axiomatic and semantic is

[1] Aristotle analyzed four modalities: possible, contingent, impossible and necessary.

accepted as a support platform (…). The modality is seen thus, as a manner or a supplementary specification of the way in which a sentence can be true"[2]. The term "modal logic" has a double meaning: (i) the limited meaning through which the alethical logic is approached; (ii) the broad meaning through which the deontic logic, the epistemic logic, the temporal logic (the chronological logic), the dynamic logic, the logic of acceptance, the teleological logic, the existential logic and so on are approached..

A distinction concerning the logical interpretation of *modalities* is necessary. Thus, if one combines the calculation of propositions with the modal logic then he/she obtains a propositional modal logic. If the calculation of predicates is combined with the modal logic then one obtains the modal logic of predicates.

G. H.von Wright distinguishes the following modalities:

(1) alethical: necessary, possible, impossible, contingent;

(2) deontic: obligatory, permitted, forbidden, indifferent;

(3) epistemic: verified, admissible (non falsified), disproved (falsified), indecisive (undecided);

(4) existential: universality, existence, nonexistence, presence and absence of a property

Among those who contributed to the development of the modal logic we can mention: C. I. Lewis, Robert Feys, G. H. von Wright, von Halden and so on. The modal logic amasses a rather consistent series of formal systems. Connecting different modalities (for example, *necessary* and *possible),* the systems of modal logic emphasize the relations specific to the logical square of oppositions. "The logical systems provided with these modal relations are usually regulated which means that by eliminating the modal functors from the theses of the modal system we find again the theses of the classical logic"[3]. In other words, in a modal system we can find rules which are specific to the traditional logic (to which a series of modal axioms is added). The spreading of the modal logic and its methods has also acquired relevance through the appearance of a semantics based on the notion of the possible world. On the other hand, there are known modal logics which through diverse applications have contributed to the appearance and development of new fields specific to scientific theories.

[2] Cornel Popa, *Logic and Metalogic (Logică şi metalogică)*, Volume II, Fundația România de Mâine Publishing House, Bucharest, 2002, p. 244.

[3] Petre Botezatu, *Logico-philosophical Interpretations (Interpretări logico-filosofice*), Junimea Publishing House, Iaşi, 1982, p. 150.

TYPES OF MODAL LOGICS SYSTEMS

One can mention among the main systems of the modal logics the following:

G.H. von Wright's modal logic (1916-2003) emphasizes two equivalent axiomatic systems (*the P system* and *the O system*): an axiomatic system where the primitive operator is permission (P) and the defined operators are obligation (O) and interdiction (F) and an axiomatic system where the primitive operator is obligation (O) and the defined operators are interdiction (F) and (P) [4].

Jan Lukasiewicz's modal logic (1878-1956) in which the Polish logician shows that modalities cannot be analyzed within a bivalent system of logic. From this point of view, starting from a bivalent logic [where are used only the values *true* (1) and *false* (0)], he succeeds in developing a structure of a trivalent logic [where he adds to the traditional values the value *possible* $(\frac{1}{2})$[5]]. Jan Lukasiewicz's modalities have been afterwards researched by Clarence Irwing Lewis and C.H. Langford who introduced the "doubtful" functor marked with D. Through the "doubtful" functor one can formulate *the principle of the excluded quart* (*quartum non datur*), according to which any proposition *p* can have one of the values 1 (true), 0 (false) or $\frac{1}{2}$(possible) which in the Lukasiewiczian symbolism can be formulated like this: "Any proposition **p** is either necessary or doubtful or impossible" [6].

Grigore Moisil's modal logic (1906-1973) where on the basis of symbolizations specific to Jan Lukasiewicz's system another system is built (1942) at its foundation one can find the operator S which is read ("maybe without").

The structure of a modal proposition is given by *dictum* and *modus*:

(a) Dictum (which is noted with **D**) – comprises the basic information;

(b) Modus (which is noted with **M**) – comprises supplementary information about the opinions, the feelings, the attitudes of the subjects of knowledge.

For example:

It is possible to snow tomorrow.

where:

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[4] Cornel Popa, *quoted works*, p. 324.

[5] **M** represents the initial from the German word "möglich" = "possible" (examples: Mp $\xrightarrow[citeste]{se}$ (IT IS READ)

"p is possible"; NMp $\xrightarrow[citeste]{se}$ (IT IS READ),"p is not possible").

[6] Anton Dumitriu, *The History of Logic (Istoria logicii*), Vol. 4, Technical Publishing House, Bucharest, 1998, p. 199.

Modus- "it is possible"

Dictum – "to snow tomorrow"

In modal logic there is a distinction between two types of sentences. In fact, this distinction becomes concrete as far as the linguistic expressions of modal propositions are concerned. It is about the distinction between those sentences where the modal word is attached to the copula (for example: "Any divisor of 12 is necessarily a divisor of 60")[7] and those sentences where, on one hand, the modal expression is prefixed to a whole sentence (for example: "Some people from this room might know something about the modal logic") and on the other hand, the modal expression is postfixed to the proposition itself (for example: "To get to that certain street you have to go round that park, it is necessary to do this")[8]. The value of truth of the modal propositions depends on the value of truth of the dictum and the truthfulness of the modus.

Examples:

-the proposition "It is necessary for students to pass their final exams to apply for a university" is a true one;

-the proposition "It is impossible to think if he/she does not have a finger at his/her left hand" represents a false one.

1.2 Forms of modal logic

1.2.1 THE DEONTIC LOGIC (OR THE NORMS LOGIC[9] OR THE IMPERATIVE LOGIC OR THE LOGIC OF DUTY)

The deontic logic ($\tau \grave{o} \delta \acute{\varepsilon} o \nu$ = to déon = "duty", "obligation") is the one that uses operators specific to its field in prescriptive (normative) reasonings. The deontic logic is a logic of obligations that has a practical character. Among those that can be considered forerunners of the deontic logic one can mention Aristotle ("Nichomachica Ethics" and "the Motion of Animals"), Martinus Schickhardus ("Judiciary Logic" - 1615), Ernst Mally, L. Lapie, Karl Menger, W. Dubislaw, Alfred Hofstadter, J. Jorgensen, Rose Rand, A. Hofstadter, Alf Ross, A. Ledent, K. Grelling. Of course one should not forget the contribution of philosophers well-known in the history of thinking such as T. Hobbes (who in the paper, "Leviathan"

[7] The sentences where the modal component is inside the proposition are called modal propositions *sine dicto* (without a distinct dictum).

[8] The enunciations where the modal expression is prefixed or postfixed to the proposition are made of an assertory proposition (*de dictum*) and a proposition that shows the modal nature of its content (*de modus*). If these two components are separated in the linguistic formulations, the latter being applied to the former as a whole then those certain enunciations represent modal propositions *cum dicto*.

[9] Another sense equivalent to the logic of norms is also *the logic of normatives*.

conceives a moral philosophy like a science on *natural laws*), B. Spinoza (who builds a logical system of the ethical norms within which the sentences are *moreo geometrico* demonstrated), John Locke (who speaks about the empirical origin of the moral norms), David Hume (who argues the relationship between the enunciative propositions and the prescriptive ones), I. Kant (who makes a classification of the norms and makes a connection between the deontic categories and the modal ones), J. S. Mill (who in the paper "The System of logic" analyses the logic of practice to which are subordinated the moral and the practice). But those who really dealt with the first deontic calculations (1950) are the Finnish Henrik von Wright[10], the French G. Kalinowschi[11], E. Garcia Máynez[12].

The deontic logic allows through the instruments that it offers problematic approaches with certain philosophical significances: "Among these one can mention the discussion of the principle of universalization (when something is obligatory/permitted/prohibited then it is like this for all) and the formal treatment of Hume's law referring to the dichotomy between facts and values, due to which from descriptive propositions it is not possible to derive any other normative proposition"[13].

The main deontic modalities are: obligatory, permitted, forbidden, indifferent.

Therefore, if one takes the word PURPIREA, then it will represent the equipollence of the following forms:

- the first vowel "U" expresses the formula *not permitted non-p (it is not permitted for S to be non-P)*

- the second vowel "I" expresses the formula *it is not obligatory p (it is not obligatory for S to be P)*

- the third vowel "E" expresses the formula *it is forbidden non-p (it is forbidden for S to be non-P)*

- the fourth vowel "A" expresses the formula *it is obligatory p (it is obligatory for S to be P)*

[10] Henrik von Wright analyses the first ideas concerning the deontic logic in works such as "Deontic logic", *Mind*, 1951, "Norm and action", 1963, and "An essay in deontic logic and the general theory of action", 1968.

[11] G. Kalinowski created two systems of deontic logic in "Théorie des propositions normatives", 1953.

[12] E. Garcia Máynez built a system of judiciary logic in the work "Los principios de la ontologia formal del derecho y su exprésion", 1953.

[13] Cosma Luminița, Pop Mihaela, Dumitru Anca (trans.), *The Encyclopedia of Philosophy and Human Sciences (Enciclopedie de Filosofie şi Ştiințe Umane*), All Educational Publishing House, Bucharest, 2004, pp. 609-610.

The deontic logic reduces itself to the modal logic. From this perspective, one can mention Alan R. Anderson's reductionist conception. Thus in the paper *The formal analysis of normative system* (1956) Alan R. Anderson shows that the deontic logic is reduced to the modal logic with the help of the notion "sanction" (not completing a fact does not necessarily imply the sanction).

This problem was also dealt with by A. N. Prior in his work "Escapism". Also, in deontic logic one speaks about the existence of some paradoxes that reflect certain limitations of applying the logic formulae on the cases specific to this type of logic[14]. From this perspective, one can mention the following deontic paradoxes:

(1) Arthur N. Prior's first paradox ("the paradox of the derived obligation") which is known with the formulation "if *p* is forbidden then if *p* is obligatory, *q* is too" and which has the formula $Fp \rightarrow O(p \rightarrow q)$, which is read "if *p* is forbidden then if *p* is obligatory, *q* is too";

(2) Arthur N. Prior's second paradox ("the good Samaritan's paradox) – which is known under the formulation "if *p* is forbidden then its conjunction with a certain *q* is also forbidden" and which has the formula $Fp \rightarrow F(p\&q)$;

(3) Alf Ross's paradox[15] - which is known with the formulation "if it is obligatory *p*, then it is obligatory *p* or *q*" and which has the formula $Op \rightarrow O(p \vee q)$ (for example: "if it is obligatory to mail a letter, then it is obligatory to mail the letter or burn it").

Another deontic system of major importance in the modal logic is *Smiley-Hanson's system*[16], which has as main characteristic the demonstration of some theorems with repeated deontic functors.

The formalization of the judiciary reasoning is possible through the logic of norms which belongs to the deontic logic. A normative proposition is neither true nor false; it can either be or not be rational.

A peculiarity of the deontic logic is the judicial logic, a term which in the reference literature has two meanings:

- a limited one- where one approaches the logic of lawful norms;

[14] Gheorghe Enescu, *Logic Dictionary (Dicționar de logică*), Scientific and Encyclopedic Publishing House, Bucharest, 1985, pp. 266-267.

[15] Alf Ross (1899-1979) is considered a philosopher of law of Danish origin.

[16] In fact, through Smiley-Hanson's system one understands the totality of those systems of deontic logic resulted from the researches done by T. J. Smiley and W. H. Hanson. From this perspective Lennart Åqvist distinguishes 10 classes of such models ("subsystems"): OK system, OM system, OS-4 system, OB system, OS-5 system, OK^+ system, OM^+ system, $OS4^+$system, OB^+ system, $OS5^+$system.

- an enlarged one- where one approaches the logic of the judicial norms and the logic of argumentation from the judicial field.

When the rational analysis concentrates on a judicial logic, in fact, one approaches the majority of the problems of formal logic (terms/notions; inductive and deductive reasonings and so on). This situation expresses a relationship between the deontic logic and the researches connected to the deontic syllogistic (Z. Ziemba), the semantics of deontic logic (S. Kripke, S. Kanger, J. Hintikka). Also, within the judicial logic there are added different postulates specific to this field. Among these postulates one can mention: (i) *nulla poena sine lege* (*there is no punishment without law*) and (ii) *nullum crimen sine lege (there is no offence without law).*

In some reference works one tried to put the basis of the deontic logic on "the theory of the actionable modalities or the human possible related to the human situations and the agents' abilities and the logic of accepting as a theory of the value judgments accepted by an agent, a group or a community"[17]. Thus, there are visible certain reevaluations concerning some classical systems and also some systems of the modal logic. Introducing some agents and taking into consideration their characteristics and implementing at the level of such course of action the concept of "actionable situation" (described through some descriptive true propositions) one does nothing but offer a perspective centered on making the modal logic pragmatic. It fact, it is about a specific program through which there are offered models applicable in the socio-human sciences and the field of artificial intelligence. In this way, one tried to build semantic theories on trees for the dynamic deontic logic and for the deontic logic referring to the states resulted from the human conduct.

1.2.2 *THE TEMPORAL LOGIC (OR* THE TIME LOGIC *OR* THE CHRONOLOGICAL LOGIC *OR* THE CHANGE LOGIC[18])

The temporal logic is the one that applies in the study of reasonings made of propositions that have a temporal aspect. Among the temporal modalities (operators) one can mention: "always", "at least", "sometime", "sometimes", "before", "after", "simultaneous", "the latest", "earlier", "now", "until" and so on.

[17] Cornel Popa, *quoted works*, p. 359.

[18] Temporal logic is studied by G.H. von Wright as a logic of change: "(...) any transformation of state can be regarded as a function of truth of some elementary state transformations (…) the p-expressions could be named in a general sense the *state descriptions* and the T-expressions as *change descriptions*", in G.H. von Wright, *(Norm and Action) Normă şi acţiune*, Scientific and Encyclopedic Publishing House, Bucharest, 1982, pp. 34 – 51.

The problem of temporal enunciations was dealt by thinkers such as: Aristotle[19], Diodor Cronos[20], William of Shyreswood (1190-1249), Heinrich Gustav Reichenbach (1823-1889), Arthur N. Prior (1914-1969)[21], Nicholas Rescher[22], G.H. von Wright (who introduces the temporal quanta of the type "always", "sometimes").

The temporal logic[23] (or the time logic or the chronological logic) deals with the temporal enunciations, in other words those propositions where the content depends on the way in which a state of fact that these express, can set in time. Consequently, "the combination of logic categories with time gives birth to some systems whose study is profitable and contributes to a better understanding of the notions connected to the concept of time (such as the verbal flexions and the verb tenses), of the relationships between modalities and time and the structure of time and the nature of negation."[24]

Also, the temporal enunciations can be closed or open. The closed temporal enunciations are those in which when one indicates the exact temporal moment of a state of fact ("in the past", "in the future"), moment that can be found in a specific chronology, thing which is possible through a process of temporal quantification. (examples: "In the past people lived better", "One can travel in the future with the help of thought", "Romanians have gained their liberty and dignity in December 1989" or "It always snows in Vatra-Dornei" or "It sometimes rains at the seaside"). The open temporal enunciations are those which use words such as: tomorrow, the day after tomorrow, today, yesterday (examples: "I sit for an exam at Maths tomorrow" or "The day after tomorrow we are going on a trip".)

The temporal logic appears in an axiomized form in Arthur N. Prior's work "Time and Modality" (1957). In fact, a long time ago Diodor interpreted the necessity in temporal terms when he says that "The necessity is what is true and what will always be true". Also in the work "Past, Present and Future", Arthur N. Prior uses the Polish marking belonging to Jan Lukasiewicz and takes as a starting

[19] At Aristotle this problem appears when the future contingents are analyzed.

[20] Diodor Cronos interpreted the necessity in temporal terms when he says that "The necessity is what is true and what will always be true", from this perspective he explains the implication according to time.

[21] Arthur N. Prior, *Time and Modality*, Oxford University Press, Oxford, 1957.

[22] Nicholas Rescher analyses the problem of temporal logic in chapter XII, "Chronological Logic" from Topics, p. 196-228, from *Philosophical Logic*, D. Reidel Publ.Comp., Dordrecht, 1968.

[23] In some specialty works there is a distinction at a conceptual level between the temporal logic and the time logic. Thus, the temporal logic is regarded as a logic from the time perspective and the time logic is the time from the logic perspective.

[24] Newton Da Costa, *Classical and Neoclassical Logics (Logici clasice şi neclasice*, Technical Publishing House, Bucharest, 2004, p. 179.

point the Wrightian modal systems when he proposes a Diodorean system of temporal logic.

The temporal logic relates to the temporal intervals[25] and the temporal relations[26] ("before", "after", "simultaneous"). Thus, one can admit the existence of a differentiated temporal logic in accordance with the adopted structure of time: circular, linear, linear time logic with beginning or linear time logic without beginning ($K_l^{\infty -}$), linear time logic with ending, linear time logic without ending ($K_l^{\infty +}$), linear time logic without beginning and without ending ($K_l^{\infty \pm}$), dense linear time logic [27] (K_{ld}), continuous time logic (K_{lr})[28]. All these forms of temporal logic can generate in certain systems concrete (valid) inferential forms. However, a specific axiomatization is not possible yet, for the time being, in some temporal structures. At the same time, one should remember that, if the temporal propositions comprise a variable part then one can say about them that they represent "time functions". Moreover, the temporal functions can be quantified ("$\forall_t$"–, "no matter the time"; "$\exists_t$" – "there is a time in which").

Met within some specific temporal systems (metrical, additive, causal, relativist, ramified, linear, rational, integral, real[29]), this type of logic (temporal logic) implies differentiation criteria which situate the operational analysis within the logical formalisms (the temporal logical system which imposes modalities specific to the grammatical time[30], the temporal logic system which imposes modalities ordered on the axis anterior-contemporary-simultaneous[31], the temporal logical system of dating[32]). There are obvious in this case the different temporal systems through which the used type of modality acquires pragmatic valences[33].

Therefore, the temporal logics come:

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[25] *The logical theory of intervals (Teoria logică a intervalelor)* represents a branch of temporal logic.

[26] The logical analysis of these relationships is analyzed by modern physics, an example from this perspective being the W. Heisenberg's uncertainty relationships.

[27] The density formulae are analyzed by the logic of rational time ($K_{ld}^{\infty \pm}$).

[28] The continuity formulae are analyzed by *the real time logic*.

[29] Petru Ioan, *Logic and Metalogic (Logică şi metalogică*), Junimea Publishing House, Iaşi, 1983, p. 159.

[30] The modalities specific to the grammatical time logic are: F_n = in the future; P_n = in the past; F = sometimes in the future; P = sometimes in the past; G = always in the future; H = always in the past.

[31] The modalities specific to such temporal logic (which are ordered in an anterior-contemporary-simultaneous way) are: T = in the next moment; Y = in the anterior moment;

[32] The modality of the temporal logic system of dating is given by a predicator of temporal making R_t, which in the form Rt (p) has the meaning: "at the t moment, p takes place".

[33] Florentina Călmăţuianu, *Typologies of the deductive systems (Tipologii ale sistemelor deductive*), "Alexandru Ioan Cuza" University Press, Iaşi, 2006, p. 208-219.

- from formulae which are valid independently from the order structure of time (the group of these formulae is known in the specialty literature as having the name "minimal temporal logic");

- by adding formulae that correspond to some possible properties of the ***R*** order structure of time in accordance with some specific formulae;

- from formulae that correspond to linearity, density and continuity etc;

- by non-admitting some specific formulae.

However, the logical context emphasizes the fact that the term "temporal logic" reminds of different meanings such as: (i) Linear Temporal Logic; (ii) Temporal Logic of Action - TLA; (iii) Branching Logic.

Also there is a distinction between temporal logic (which is logic from the time perspective) and tense logic (*tense logic* – a logic through which the tense is analyzed from a logical perspective). Arthur Prior introduced *tense logic* to explain the idea of logical time from the logical perspective. In the same way, Leslie Lamport introduced *temporal logic* of actions in order to make easy the checking of some systems. From this point of view there has been introduced a language for the temporal logic of actions known as TLA^+.

The universe of the temporal logic is built on the theory of Zermelo-Fraenkel (ZM) groups. The temporal logic found applications in formal checkings [(i) *model checking*[34] – which supposes an exhaustive systematic exploration of the mathematical model; the aimed aspects from this perspective being those connected with *linear temporal logic* (LTL)[35] and *computational tree logic* (CTL)[36]; (ii) of the *logical inference* type – where there are aimed problems of understanding some softwares of formal validation of the mathematical reasoning (HOL Theorem prover[37], ACL2[38], Isabelle[39] or Coq[40]], through which there can be demonstrated certain specific theorems.

[34] Web address: http://www.cs.utt.ro/~marius/curs/vf/curs2.pdf, accessed 22nd August 2009.

[35] Amir Pnueli (n. 1941) and Zohar Manna (n. 1939) dealt with this field in works such as "The Temporal Logic of Reactive and Concurrent Systems: Specification", Springer-Verlag, 1991; "The Temporal Logic of Reactive and Concurrent Systems: Safety", Springer-Verlag, 1995; "The Temporal Logic of Reactive and Concurrent Systems: Progress" (the preparing stage).

[36] C*omputational tree logic* (CTL) is in fact a ramified logic of time. This field was particularly dealt by Edmund Clarke and E. Allen Emerson.

[37] *HOL Theorem* prover is a program that allows "the demonstration" of theorems.

[38] *ACL2* (*A Computational Logic for Applicative Common Lisp*) is software system which consists of a programming language, a theory extended in fact, to the first level of the formal logic (the predicates logic, of the first order, the logic of predicates, of the-n order).

[39] *Isabelle* is HOL Theorem prover's successor.

[40] *Coq* allows the checking of mathematical expressions (assertions) (web address: http://coq.inria.fr/about-coq, accessed 2nd September 2009).

Also *the temporal intervals logic* (which is a specific form of temporal logic) developed by Ben Moszkowski, found its usefulness in the formal description of the hardware and software systems.

The formation of the temporal enunciations reminds, on the one hand, of their defining from a temporal point of view (their truthfulness and falsity do not take into account the time of assertion), and on the other hand, of their temporal non defining (the truthfulness or the falsity depend on the time of assertion). This problem can also be found in the interest sphere of the logician Petru Ioan when he analyses the formalization of the enunciations temporally affected[41]. Thus, at the centre of these researches there is the fundamental concept of temporal logic which is called *temporal accomplishment* expressed through the following formula:

R_t (a) = A is accomplished at the time t

In this way, there results the construction of the system R of temporal logic as an extension of the common logic.

1.2.3 THE EPISTEMIC LOGIC

The epistemic logic is a specific logic through which one tries to explain "those in tensional contexts of belief and knowledge"[42]. Consequently, in the reference literature there was proposed an *epistemic logic* (different from *the logic of belief*) in the true sense of the word as soon as Jaakko Hintikka's work *Science and Belief* (1962) was published. Specific to these two types of logic is the fact that both remind of a semantic of the possible worlds[43]. Therefore, within an epistemic logic we study propositions where the modalities of knowledge *to know* and *to believe* are obvious, which are in fact, interdefinable/irreductible (a property resulted from the fact that those certain modalities represent expressions of the knowledgeable subject's attitudes.) This fact allows at a theoretical level a distinction between "*a stricto-sensu epistemic logic* (of knowledge) and a *doxastic logic* (of opinion)" [44].

Starting from the note B*ap* (*a* thinks that *b*)[45] and K*ap* (*a* knows that *p*)[46] one

[41] Petru Ioan, *Logical Perspectives (Perspective logice*), Junimea Publishing House, Iaşi, 1987, pp. 252-253.
[42] Marie-Dominique Popelard and Denis Vernant, *Elements of logic (Elemente de logică)*, European Institute, Iaşi, 2003, p. 100.
[43] In logic the moment one talks about *a semantic of the possible worlds*, one admits that the logical laws represent enunciations valid in all possible worlds.
[44] Florentina Călmățuianu, *quoted works*, p. 220.
[45] The "B" letter comes from, "Believe".
[46] The "K" letter comes from "Know".

can make a calculi database which supposes the existence of the following theorems:

$Kap \supset KaKap \xrightarrow[citeste]{se}$ IT IS READ If *a* knows that p, then *a* knows that *a* knows that *p*

$Bap \supset KaBap \xrightarrow[read]{it's}$ IT IS READ If *a* thinks that *p*, then *a* knows that *a* thinks that *p*

$Kap \supset BaKap \xrightarrow[read]{it's}$ IT IS READ If *a* knows that *p*, then *a* thinks that *a* knows that *p*

As a result, the laws of the scientific contexts specific to the epistemic logic[47] are characterized though the epistemic in tensional operators. Their role consists in the fact that they express modalities that refer to the cognitive relationship.

Examples of epistemic in tensional operators: "it is known that"(knowledge operator), "it is really thought that", " it is supposed that", (belief act operators), "it is demonstrable that", "it is reasonable to think that" (operators specific to a logic of justifying or founding).

Newton da Costa shows that the epistemic logic sets on the one hand within the category of the thetic systems and on the other hand in the category of the non-thetic systems of the second species[48] (there are formulated hypotheses and suppositions which cannot be considered in the true sense of the word as true or false but are rather considered systematizations of some results obtained as a consequence of the experimental researches).

Within this epistemic logic one can find the autoepistemic logic introduced by Moore and completed by Stalkner. Built on the basis of the propositional modal logic, the autoepistemic logic has as a specific form of language the introspective modal operator ***L***. for instance, a formula **L***a* is read" it is believed that *a* is true". A variant of the autoepistemic logic is *Nonmonotonous modal logic N* (also called *the pure logic of necessity*) which is considered a *weak logic* because it does not contain its own modal axioms.

[47] The importance of the epistemic logic is relevant in the theory of knowledge, an example from this perspective being the one which reminds of Gödel's theorems through which it is shown that no form of knowledge can offer an unconditional guarantee of truth.

[48] The non-thetic systems of second species are part of the heterodox (nonclassical) logic system which is that logic where there are not satisfied at least two of the conditions of existence specific to the orthodox (classical) logic.

1.2.4 THE EXISTENTIAL LOGIC

Existential logic has as existential modalities: the universality, the existence, the partiality (the presence of a property) and the void. That is why, the logic of acceptance is regarded as a logic of the value judgments.

1.2.5 THE TELEOLOGICAL LOGIC

The teleological logic is a logic of goals and subordinates to a logic of human actions. Also, this type of logic combines perfectly with elements of *the logic of preference.* Built on analogy and not on an extension of the deontic logic, the teleological logic (the teleo-logic differs from the deonto-logic) comes to justify the necessity of the concept of "goal" at the level of a complex relationship assumed between certain logical entities. Thus, one can find at the level of a teleological logic formulae such as S(a, t_1, p) and S (a, l, p) which are read "agent *a* proposes at the moment t_1 as goal the state *p*" and "agent *a* wants to reach the state *p* through *l*", formulae which can be found in a unitary relationship of the type S(a, t_1, p, t_2, q, l) which is read "agent *a* proposes at the moment t_1 and in the conditions *p* to achieve at the moment t_2, the state *q* through the series of acts *l*".

One can notice in this case a connection between the teleological logic and the dynamic one where it appears as a fundamental idea of a pragmatic nature which is given by the relationship goal-conduct.

The respective agent is in other words in an actionable situation and being based on a choice it adopts a certain conduct to reach a goal. In these conditions, the goal as a form of concretization supposes a sequence of conducts through which a decision is taken.

1.2.6 THE DYNAMIC LOGIC

The dynamic logic represents a type of modal logic which reminds of the modalities of human action. That is why this type of logic has been positioned together with the logic of acceptance alongside the deontic logic or even the teleological logic in a general logic of action.

V. R. Pratt is the one who put the basis of the dynamic logic focusing on some former researches accomplished by R. M. Floyd and C. A. R. Hoare. EPDL – *elementary propositional dynamic logic* is given by an alphabet specific to the atomic formulae, an alphabet specific to the atomic programs and certain concepts derived from some definitions. At the same time, the semantic construction of this type of logic has been possible on the basis of the theory of the possible worlds which was promoted by Saul. A. Kripke and on the basis of some researches done by Jakko Hintikka and Stig Kanger.

The applications of such type of dynamic logic can be found in the making of some logical programs where the component operations (from the virtual world) belong to the agents (from the real world). Thus, in the terms of the dynamic logic one can describe both the workings of household appliances (the washing machine, the cooker, the microwave oven) and daily actions specific to the different agents (taking into account the criteria of age, weight, sex, profession etc.). For instance, before an exam the teacher proposes to his/her students more ways of taking the exam (written, oral, both written and oral) and then taking into consideration what has been negotiated a conclusion has been reached. In the language of the dynamic logic one admits that it was about the operation of choice meaning that was preferred a subgroup x_1 (written exam) from the general group *X* (written exam, oral exam, oral and written exam). All in all, through some expert and artificial intelligence systems but also with the help of an algorithm which uses operations specific to the dynamic logic (first or *n* order), different social and technical problems can be solved. Consequently, the necessity of a dynamic logic becomes major as long as the latter subordinates positively speaking to a logic of (human) action.

1.2.7 THE LOGIC OF ACCEPTANCE

The logic of acceptance reminds of the idea of argumentation in favor of a logical object. In other words, everything that is given arguments must be accepted too. The appearance of a logical theory of acceptance was caused by the need to build a syntactic, semantic and pragmatic model of a certain argumentative course of action. From this perspective, the analysis of a logic of acceptance is done by Cornel Popa[49] when he shows that it represents the Cartesian product: Ag x W x D (where Ag- the group of agents, W- the group of the actionable situations or of the possible worlds, D –any of the subfields D1-D17). Among these subfields one can enumerate: the world of opinions, proposals, value judgments, offers, orders, advice, excuses, decisions in a certain field, theories, solutions to some theoretical or practical problems etc. The modalities with which the logic of acceptance operates are: "accepted", "rejected", "doubtful" ("doubt"), "accepts with full conviction", "rejects with full conviction".

Thus, there are emphasized two variants specific to the logic of acceptance:

(i) the trivalent variant (V1) = {a, r, d}

[49] Cornel Popa, "The logic of acceptance, the opinions and the argumentation" (Logica acceptării, opiniile şi argumentarea), in *"Spiru Haret" University Annals,* Studies of Philosophy Series, No. 3, 2001, Fundația România de Mâine Publishing House, Bucharest, 2001, pp. 61-78 and in Cornel Popa*, Logic and Metalogic (Logică şi metalogică)*, Volume II, Fundația România de Mâine Publishing House, Bucharest, 2002, pp. 394-429.

where: a – "accepts";
r – "rejected";
d – "doubtful" ("doubt").

(ii) the pentavalent variant (V2) = {a, r, d, c, s}
where: a – "accepts";
r – "rejected";
d – "doubtful" ("doubt");
c – "accepts with full conviction";
s – "rejects with full conviction".

The evaluation of the two variants specific to the logic of acceptance supposes a perspective meant to justify the necessity of a new type of axiomatization at the level of the possible attitudes. Thus, the acceptance of the logical laws specific to the theory of acceptance allows the agents (Ag) to accept or tolerate them. To sum up, those certain agents' behaviour is one of a dynamic type and depends on the reference system where they show their own attitudes. Consequently, the theory of acceptance is a world of the human conduct where the acceptance (rejection) determines a certain action. From this perspective, different variants of logic of acceptance have been built by relating to certain actionable situations.

1.2.8 THE LOGIC OF ACTION

The logic of action comes from the manifestation of the logic of acceptance. Moreover, in the context of the new formalizations the logic of action includes the deontic logic, the teleological logic, the logic of preferences and the theory of decisions. The logic of action is visible exactly through the introduction, at the level of a logic of acceptance, of some actionable situations for each agent (Ag) involved in an argumentative-pragmatic course of action. Thus, in the field of the logic of action one can find branches of logic that remind especially of the human practical activities.

However, when one admits that the analysis of the human acts generates the appearance of a logic system it is about, in fact, a "logic in action" and not a "logic of actions" (which has to do with the field of praxeology). In this context, Petre Botezatu states that one has to avoid the confusion that could arise between the fact of thinking of logic as a theory of action (the genetic logic) and the logic understood as a theory of the efficient action (praxeology)." The genetic logic is a theory of action in a completely different meaning to praxeology. It has to do with the logical system which can be taken out from the analysis of the human acts: this is in fact *the logic of action*, but not the *logic of actions* which is in the competence

of praxeology".[50] However, logic and praxeology although they stand as distinct fields of analysis, fundament each other circularly, meaning that logic supposes praxeology and praxeology supposes logic.

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[50] Petre Botezatu, *The creation of logicity (Constituirea logicității*), Scientific and Encyclopedic Publishing House, Bucharest, Bucureşti, 1983, p. 85.